Math151 - Fall 2007 Extra Credit Project – Louis Gross

This project is not required for the course - if you choose to complete this project it will only be applied to increase your course grade. If its effect would be to decrease your course grade, it will be ignored. It counts in the overall grading as an additional project, thus it will count a maximum of 3% of your overall course grade. You are required to carry out the work on this project individually, not in a group.

Finding the Maximum Harvest in a Simple Population Growth Case

This project assumes that a population of organisms (think of them as fish in a lake) grows according to a very simple rule: each time period, the population increases by one individual with probability p and decreases by one with probability 1-p. We will assume that the population is harvested when it reaches a particular number (b) of individuals, and the fraction of the population harvested at that time is f. Thus the amount the population is reduced by harvesting is f*b. Further we assume that if the population size drops too low, it will no longer be considered feasible to harvest (below a minimum viable population size which is set at .1* the initial population size), and any harvest obtained in such a population is ignored (e.g. there is a large penalty of getting no harvest at all if you reduce the population below its minimum viable size). We will assume that the starting population size is 100 individuals and that this harvesting operation goes on for a fixed time period (300 time periods) and then stops. So the absolute maximum number of individuals in the population that could be obtained from harvesting is 400.

Your challenge is to investigate how large b can be made for different values of p, to provide the largest mean harvest possible while maintaining the population size above the minimum viable level. You will do this for a particular fixed value of f, that is chosen according to your birthdate as:

.6 + ((Your age in years)/200) + ((The number of your birthmonth)/100). So if you were born in July of 1980 (so you are now 25 years old) you would choose f = .6 + 25/200 + 7/100 = .795

For your particular value of f, do the following and create a typed report (no handwritten report is acceptable):

(1) For p varying from .5 to .9 by units of .1, find the maximum mean total harvest and the standard deviation of maximum total harvest and the b value used to obtain these, providing a table of your results. In your report, describe the method you used to vary the range of b values in order to obtain the maximum harvest.

(2) Plot the values obtained in (1) as a function of p showing in the graph the b value at which maximum mean harvest occurred, and the mean and standard deviation of the maximum harvest you obtained. Include a legend for this figure that describes

the components (e.g. what symbols you used for each item plotted and what line style you used to connect them)

(3) Interpret your results by (a) describing the relationships you obtained in a few sentences, (b) noting how the choice of b affected the harvest obtained for any particular p-value, (c) what you expect to happen when p is smaller than the values you used (e.g. below .5), (d) what are the limitations in using the results of this for harvesting of real-world populations, and (e) what if any general conclusions can you draw from the analysis and how might these change if you changed the assumptions in the experiment. .

NOTE: I have provided a Matlab program, harvestopt.m, to assist you in carrying out this project. This requires you to enter your value of f, the p value, the number of simulations n to be done for each b value (I suggest you set this to at least 30), and the lower and upper values for b to use. The program automatically divides the range of b values you gave into 10 equal sections and calculates the mean and standard deviation of total harvest as well as the mean and standard deviation of the number of harvests at each of the b values. In using this, for each p value be sure to vary the range of b values so as to make sure you find the maximum. The program graphs first the mean and standard deviation of the number of harvests as a function of the b values and then the mean and standard deviation of the total amount of harvest as a function of the b values.