

Math 152 - Sample Final Exam - Answers (con'd.)

4. (a)  $K = 150$  tons/hectare (b)  $B'(a) = \frac{13500 e^{-a/10}}{(10 + 90 e^{-a/10})^2}$

(c)  $B(a) = 75 \Rightarrow a = 10 \ln 9 = 22.0$  years

(d)  $B''(a) = r B' \left( \frac{K - B}{K} \right) + r B \left( -\frac{B'}{K} \right) = r B' \left( 1 - \frac{2B}{K} \right) = 0$  when  $B = K/2$  so  $B'$  is

maximized when  $B=K/2$ . This implies that the stand being harvested at 22 years would be harvested when the biomass growth rate has been maximized. Harvesting later than this would give a period of stand growth at lower than the maximum growth rate.

5.  $N(t) = \frac{1}{2} t^3 + \frac{t^2}{2} + 4t + 2$

6.  $\int_0^2 (4x - 2x^2) dx = 8/3$

7. (a)  $\frac{3}{4} e^{-4x} \left( x + \frac{1}{4} \right) + C$  (b)  $\frac{4}{9} = .44$  (c)  $\frac{(\ln x)^2}{2} + C$

8. (a)  $L'(0) = .2(40 - 3) = 7.4$  cm/month

(b)  $L(0) = 3 = 40(1 - e^{.2t_0}) \Rightarrow t_0 = 5 \ln(37/40) = -.39$

so  $L(t) = 20 = 40(1 - e^{-.2(t + .39)})$  which implies  $t = -5 \ln(1/2) - .39 = 3.07$  months

9.

$$\int_0^6 40(12 - x) \pi \left( \frac{-1}{4} x + 6 \right)^2 dx = 61830 \pi = 194244 \text{ kg m}$$

10.(a)  $y(t) = 4 e^{t^2 + t}$

(b)  $N(t) = c t^{1/2}$