Simple Deterministic IPM

Maggie Sullens

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Chapter idea

- A single attribute z differentiates between individuals
  - z is mainly size in this chapter
  - While z can be size, z cannot be growth rate
- The IPM gives individuals with the same state or z the same probability of surviving to the next state, reproducing, or producing a certain number of offspring (depending on what's being modeled)

The model

- $n(z,t)$ describes the state of the population at time t based on the size distribution.
  - The number of individuals with size z in the time interval $[a,b]$ is

$$\int_a^b n(z,t)dz$$

  - A more intuitive description is: "The number of individuals in the size interval $[z, z+h]$ at time t is approximately $n(z,t)h$ when h is small".

Important functions

- z: size at time t
- z': size at time $t+1$
- $P(z', z)$: survival followed by possible growth or shrinkage from time
  - $P(z', z) = s(z)G(z', z)$ where s describes the chance of survival and G describes the transition.
- $F(z', z)$ representing per-capita production of new recruits

- $P(z', z)h$ is the probability that the individual is alive at time $t+1$ and its size is in the interval $[z', z'+h]$
- $F(z', z)h$ is the number of new offspring in the interval $[z', z'+h]$ present at time $t+1$, per size-z individual at time t.

The Kernal

- The net result of survival and reproduction is summarized by the function:

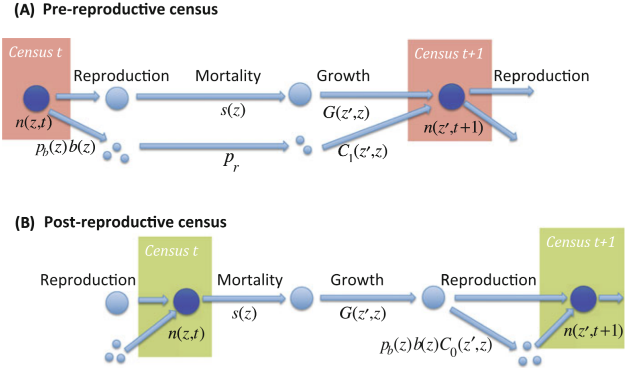
$$K(z', z) = P(z', z) + F(z', z) = s(z)G(z', z) + F(z', z)$$

- Population size at time $t + 1$ is $n(z', t + 1) = \int_L^U K(z', z)n(z, t)dz$

IPM Idea

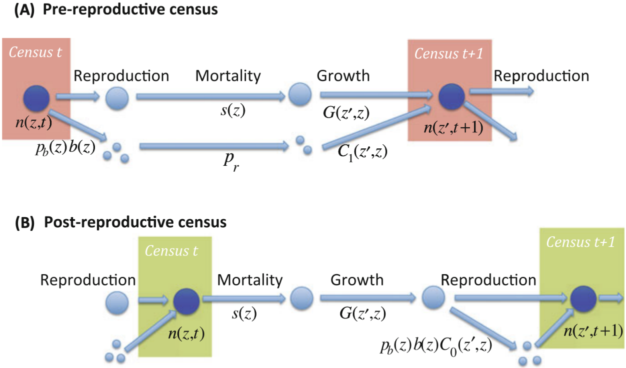
- Assume data is gathered from marking an individual and following throughout its life
  - check in with individual at evenly spaced time intervals (census)

Census placement



- $p_b(z)$:the probability of reproduction
- $b(z)$: the size-specific fecundity.
- p_r: probability of successful recruitment of each new offspring
- $C_k(z', z)$: new recruitment's size at the census k

Kernel differences



- Pre-reproductive census:
  $K(z', z) = s(z)G(z', z) + p_b(z)b(z)p_r C_1(z', z)$
- Pre-reproductive census:
  $K(z', z) = s(z)G(z', z) + s(z)p_b(z)b(z)C_0(z', z)$

Table for analysis

- Pre-reproductive Census

Size t	Offspring	Survive	Reproduced	Size $t+1$
3	NA	0	0	NA
7	5	1	1	8
8	4	1	1	10
5	NA	1	0	4

- Post-reproductive Census

Size t	Offspring	Survive	Reproduced	Size $t+1$
3	NA	0	NA	NA
7	5	1	1	8
8	4	1	1	10
5	NA	1	0	4

Building the IPM

- Pre-reproductive Census
  - Start by leaving out size and focusing on the ways individuals contribute to the next time step:
    1. Surviving
    2. Reproducing

$$N(t+1) = sN(t) + p_b p_r bN(t) = (s + p_b p_r b)N(t)$$

$$K = s + p_b p_r b \text{ (which is just a number)}$$

  - Add in the size distribution:

$$N(t+1) = \int_L^U \left(s(z) + p_b(z) p_r b(z)\right) n(z,t) dz$$

$$K(z) = s(z) + p_b(z) p_r b(z)$$

  - Lastly, forecast size distributions for next time step (add in $G(z', z)$ and $C_1(z', z)$)

$$K(z', z) = s(z)G(z', z) + p_b(z) p_r b(z) C_1(z', z)$$

Building the IPM

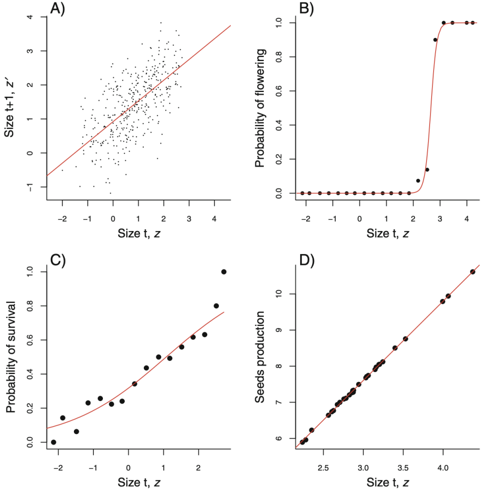
- Post-reproductive Census

$$K(z', z) = s(z)G(z', z) + s(z)p_b(z)b(z)C_1(z', z)$$

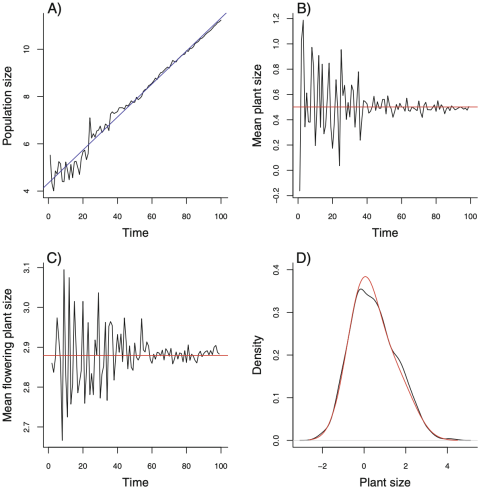
Case Study 1:Monocarpic plants

- Reproduction is fatal
- Pre-reproductive census
- Kernal: $K(z', z) = (1 - p_b(z))s(z)G(z', z) + p_b(z)b(z)p_r c_0(z')$

Case Study 1:Monocarpic plants



Case Study 1:Monocarpic plants



Soay sheep

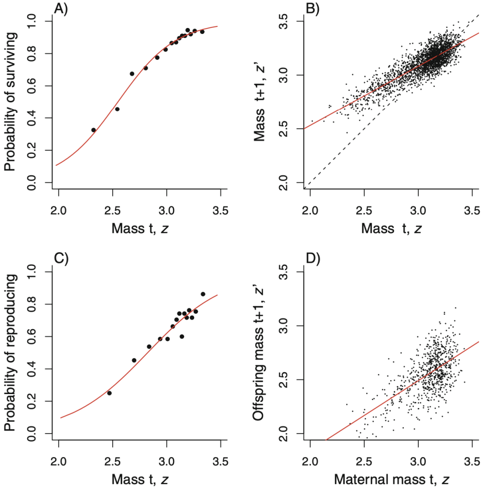
- Each year newborn individuals are caught, weighed, and tagged shortly after birth in the spring
- Each Summer, half the population's bod mass is measured
- Maternity is observed through field observation
- Population density well documented through periodic censuses and mortality searches

Soay sheep

- Census for IPM is placed in summer (post-reproductive census)
- Mortality mainly happens in winter
- Simplifying assumptions for the model:
  1. only consider the dynamics of females
  2. ignore the impact of age-structure
  3. assume that the environment does not vary between years
  4. assume that Soay females only bear singletons

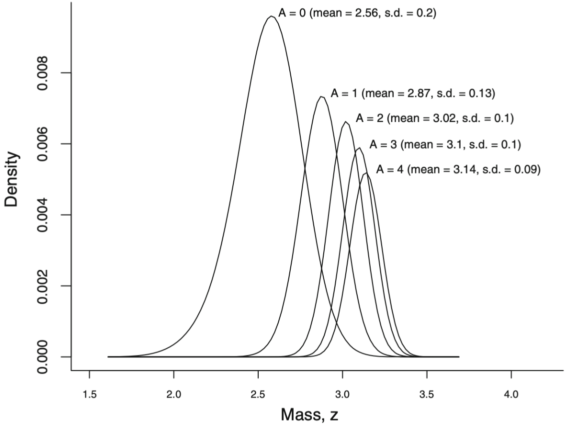
Soay sheep

- Kernal: $K(z', z) = s(z)G(z', z) + s(z)p_b(z)p_r C_0(z', z)/2$



Soay sheep

- For a sanity check, you can look at stable size-age structures:



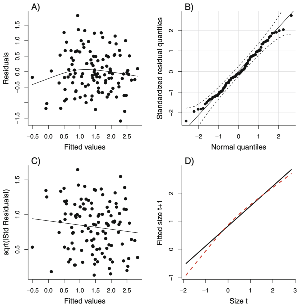
Concepts to think about

Model structure:

- Model terms should be something you can read out loud.

$s(z)G(z', z)$:
An individual of size z at time t will be size z' at time $t+1$ if it survives, and then grows (or shrinks) to size z'

- Residuals should:
  1. have constant variance and no trend in mean
  2. be Gaussian



Concepts to think about

Choose size range carefully.

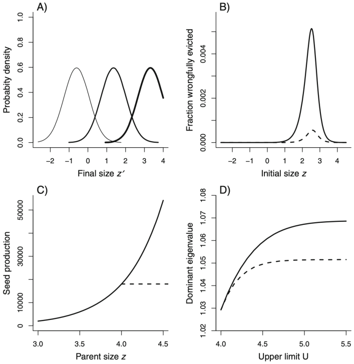


Fig. 2.11 Detecting and eliminating eviction of large individuals in the *Oenothera* IPM. A) Plots of the growth kernel $G(z', z)$ for z at the lower endpoint, midpoint, and upper endpoint of the size range $[L, U] = [-2.65, 4]$. B) Size-dependent probability of eviction with $L = -2.65$ and upper size limit $U = 4$ (solid) and $U = 4.5$ (dashed). C) Per-capita seed production as a function of parent size with no demographic ceiling (solid) and a ceiling at $U_1 = 4$ (dashed). D) Dominant eigenvalue λ as a function of the upper limit U for $L = -2.65$ with no demographic ceiling (solid) and a ceiling at $U_1 = 4$ (dashed). Source file: `MonocarpGrowthEviction.R`

Questions?

What if you wanted to use another defining factor like age in addition to size?