>

Supplementary Information for "Rapid evolution of reproductive barriers driven by sexual conflict" by S. Gavrilets

PART 1 This is a printout of a Maple notebook (available from the author and Nature)

Derivation of equations 2a and 10a from equations 9

Below I'll be using X=x-x_mean and delta=x_mean-y_mean so that x-y_mean=X+delta. Also, Y=y-y_mean.

The proportion of the males compatible with female trait y is

```
> P:=1-alpha*(V[y]+(X+delta)^2);
```

$$P := 1 - \alpha\,(V_y + (X+\delta)^2)$$

```
> expand(");
```

$$1 - \alpha\,V_y - \alpha\,X^2 - 2\,\alpha\,X\,\delta - \alpha\,\delta^2$$

```
> P:=subs(delta^2=kappa/alpha,");
```

I introduce a special notation for kappa=alpha*delta^2
because in different expansions alpha is small (<<s,theta) but kappa will have the same order as s and theta

$$P := 1 - \alpha\,V_y - \alpha\,X^2 - 2\,\alpha\,X\,\delta - \kappa$$

Female fitness function (from equation 1a):

```
> W[x] := 1-S*(P-1+theta)^2;
```

$$W_x := 1 - S\,(-\alpha\,V_y - \alpha\,X^2 - 2\,\alpha\,X\,\delta - \kappa + \theta)^2$$

coefficients a_i in fitness function expansion (8):

```
> A:=i->collect(expand(subs(X=0,diff(W[x],X$i)/i!)),alpha);
```

$$A := i \to \mathrm{collect}\left(\mathrm{expand}\left(\mathrm{subs}\left(X=0, \frac{\mathrm{diff}(W_x, X\,\$\,i)}{i!}\right)\right), \alpha\right)$$

```
> subs(Y=0,W[x]): a[0]:=collect(expand("),alpha);
```

$$a_0 := (-4\,S\,X^3\,\delta - 2\,S\,V_y\,X^2 - 4\,S\,V_y\,X\,\delta - 4\,S\,X^2\,\delta^2 - S\,V_y^{\,2} - S\,X^4)\,\alpha^2$$
$$+ (-2\,S\,X^2\,\kappa - 4\,S\,X\,\delta\,\kappa + 4\,S\,X\,\delta\,\theta + 2\,S\,V_y\,\theta + 2\,S\,X^2\,\theta - 2\,S\,V_y\,\kappa)\,\alpha + 1 - S\,\kappa^2$$
$$+ 2\,S\,\kappa\,\theta - S\,\theta^2$$

```
> A(1): a[1]:=";
```

$$a_1 := -4\,S\,\alpha^2\,\delta\,V_y + (-4\,S\,\delta\,\kappa + 4\,S\,\delta\,\theta)\,\alpha$$

```
> A(2): a[2]:=";
```

$$a_2 := (-4\,S\,\delta^2 - 2\,S\,V_y)\,\alpha^2 + (-2\,S\,\kappa + 2\,S\,\theta)\,\alpha$$

```
> A(3): a[3]:=";
```

$$a_3 := -4\,S\,\alpha^2\,\delta$$

```
> A(4): a[4]:=";
```

$$a_4 := -S\,\alpha^2$$

```
> A(5); A(6);
```

$$0$$

$$0$$

Denominator of 9a (mean fitness):

```
> Denom:=a[0]+sum(a[i]*M[i],i=2..4);
```

$$Denom := (-4\,S\,X^3\,\delta - 2\,S\,V_y\,X^2 - 4\,S\,V_y\,X\,\delta - 4\,S\,X^2\,\delta^2 - S\,V_y^{\;2} - S\,X^4)\,\alpha^2 + (-2\,S\,X^2\,\kappa - 4\,S\,X\,\delta\,\kappa + 4\,S\,X\,\delta\,\theta + 2\,S\,V_y\,\theta + 2\,S\,X^2\,\theta - 2\,S\,V_y\,\kappa)\,\alpha + 1 - S\,\kappa^2 + 2\,S\,\kappa\,\theta - S\,\theta^2 + ((-4\,S\,\delta^2 - 2\,S\,V_y)\,\alpha^2 + (-2\,S\,\kappa + 2\,S\,\theta)\,\alpha)\,M_2 - 4\,S\,\alpha^2\,\delta\,M_3 - S\,\alpha^2\,M_4$$

Numerator of 9a

```
> Numer1:=sum(a[i]*M[i+1],i=1..4);
```

$$Numer1 := (-4\,S\,\alpha^2\,\delta\,V_y + (-4\,S\,\delta\,\kappa + 4\,S\,\delta\,\theta)\,\alpha)\,M_2 + ((-4\,S\,\delta^2 - 2\,S\,V_y)\,\alpha^2 + (-2\,S\,\kappa + 2\,S\,\theta)\,\alpha)\,M_3 - 4\,S\,\alpha^2\,\delta\,M_4 - S\,\alpha^2\,M_5$$

\> Equation 9a

```
> subs(delta^2=kappa/alpha, Numer1/Denom);
```

$$\left((-4\,S\,\alpha^2\,\delta\,V_y + (-4\,S\,\delta\,\kappa + 4\,S\,\delta\,\theta)\,\alpha)\,M_2 + \left(\left(-4\,\frac{S\,\kappa}{\alpha} - 2\,S\,V_y\right)\alpha^2 + (-2\,S\,\kappa + 2\,S\,\theta)\,\alpha\right)M_3 - 4\,S\,\alpha^2\,\delta\,M_4 - S\,\alpha^2\,M_5\right) \Big/ \left(\left(-4\,S\,X^3\,\delta - 2\,S\,V_y\,X^2 - 4\,S\,V_y\,X\,\delta - 4\,\frac{S\,X^2\,\kappa}{\alpha} - S\,V_y^{\;2} - S\,X^4\right)\alpha^2 + (-2\,S\,X^2\,\kappa - 4\,S\,X\,\delta\,\kappa + 4\,S\,X\,\delta\,\theta + 2\,S\,V_y\,\theta + 2\,S\,X^2\,\theta - 2\,S\,V_y\,\kappa)\,\alpha + 1 - S\,\kappa^2 + 2\,S\,\kappa\,\theta - S\,\theta^2 + \left(\left(-4\,\frac{S\,\kappa}{\alpha} - 2\,S\,V_y\right)\alpha^2 + (-2\,S\,\kappa + 2\,S\,\theta)\,\alpha\right)M_2 - 4\,S\,\alpha^2\,\delta\,M_3 - S\,\alpha^2\,M_4\right)$$

```
> simplify(");
```

$$S\,\alpha\,(4\,\delta\,M_2\,\alpha\,V_y + 4\,\delta\,M_2\,\kappa - 4\,\delta\,M_2\,\theta + 6\,M_3\,\kappa + 2\,M_3\,\alpha\,V_y - 2\,M_3\,\theta + 4\,\alpha\,\delta\,M_4 + \alpha\,M_5)\Big/ (4\,S\,\alpha^2\,X^3\,\delta + 2\,S\,\alpha^2\,V_y\,X^2 + 4\,S\,\alpha^2\,V_y\,X\,\delta + 6\,S\,\alpha\,X^2\,\kappa + S\,\alpha^2\,{V_y}^2 + S\,\alpha^2\,X^4 + 4\,S\,\alpha\,X\,\delta\,\kappa - 4\,S\,\alpha\,X\,\delta\,\theta - 2\,S\,\alpha\,V_y\,\theta - 2\,S\,\alpha\,X^2\,\theta + 2\,S\,\alpha\,V_y\,\kappa - 1 + S\,\kappa^2 - 2\,S\,\kappa\,\theta + S\,\theta^2 + 6\,S\,\alpha\,M_2\,\kappa + 2\,S\,\alpha^2\,M_2\,V_y - 2\,S\,\alpha\,M_2\,\theta + 4\,S\,\alpha^2\,\delta\,M_3 + S\,\alpha^2\,M_4)$$

```
> "/(S*alpha);
```

$$(4\,\delta\,M_2\,\alpha\,V_y + 4\,\delta\,M_2\,\kappa - 4\,\delta\,M_2\,\theta + 6\,M_3\,\kappa + 2\,M_3\,\alpha\,V_y - 2\,M_3\,\theta + 4\,\alpha\,\delta\,M_4 + \alpha\,M_5)\Big/ (4\,S\,\alpha^2\,X^3\,\delta + 2\,S\,\alpha^2\,V_y\,X^2 + 4\,S\,\alpha^2\,V_y\,X\,\delta + 6\,S\,\alpha\,X^2\,\kappa + S\,\alpha^2\,{V_y}^2 + S\,\alpha^2\,X^4 + 4\,S\,\alpha\,X\,\delta\,\kappa - 4\,S\,\alpha\,X\,\delta\,\theta - 2\,S\,\alpha\,V_y\,\theta - 2\,S\,\alpha\,X^2\,\theta + 2\,S\,\alpha\,V_y\,\kappa - 1 + S\,\kappa^2 - 2\,S\,\kappa\,\theta + S\,\theta^2 + 6\,S\,\alpha\,M_2\,\kappa + 2\,S\,\alpha^2\,M_2\,V_y - 2\,S\,\alpha\,M_2\,\theta + 4\,S\,\alpha^2\,\delta\,M_3 + S\,\alpha^2\,M_4)$$

Dominant part (in alpha)

```
> factor(subs(alpha=0,"));
```

$$2\,\frac{2\,\delta\,M_2\,\kappa - 2\,\delta\,M_2\,\theta + 3\,M_3\,\kappa - M_3\,\theta}{-1 + S\,\kappa^2 - 2\,S\,\kappa\,\theta + S\,\theta^2}$$

Neglecting 3rd moment

```
> factor(subs({M[3]=0,M[4]=3*M[2]^2},"));
```

$$4\,\frac{\delta\,M_2\,(\kappa - \theta)}{-1 + S\,\kappa^2 - 2\,S\,\kappa\,\theta + S\,\theta^2}$$

!!!!!!!!!!!!!!!!!!!!!! Equations 2a: !!!!!!!!!!!!!!!!!!!!!!
(neglecting s and kappa relative to 1 and dividing by 2 to account for sex-limited expression of the trait)

```
> Delta[x]:=1/2*(s*alpha/theta^2)*subs({S=0,kappa=alpha*(x-y)^2
  ,delta=x-y, M[2]=V[x]},");
```

$$\Delta_x := -2\,\frac{s\,\alpha\,(x-y)\,V_x\,(\alpha\,(x-y)^2 - \theta)}{\theta^2}$$

Numerator in 9b

```
> Numer2:=a[1]*M[3]+sum(a[i]*(M[i+2]-M[i]*M[2]),i=2..4);
```

$$Numer2 := (-4\,S\,\alpha^2\,\delta\,V_y + (-4\,S\,\delta\,\kappa + 4\,S\,\delta\,\theta)\,\alpha)\,M_3 + ((-4\,S\,\delta^2 - 2\,S\,V_y)\,\alpha^2 + (-2\,S\,\kappa + 2\,S\,\theta)\,\alpha)\,(M_4 - {M_2}^2) - 4\,S\,\alpha^2\,\delta\,(M_5 - M_3\,M_2) - S\,\alpha^2\,(M_6 - M_4\,M_2)$$

Equation 9b

```
> subs(delta^2=kappa/alpha, Numer2/Denom-(Numer1/Denom)^2);
```

$$((-4\,S\,\alpha^2\,\delta\,V_y+(-4\,S\,\delta\,\kappa+4\,S\,\delta\,\theta)\,\alpha)\,M_3+\%1\,(M_4-{M_2}^2)-4\,S\,\alpha^2\,\delta\,(M_5-M_3\,M_2)$$
$$-S\,\alpha^2\,(M_6-M_4\,M_2))\bigg/\Bigg($$
$$\left(-4\,S\,X^3\,\delta-2\,S\,V_y\,X^2-4\,S\,V_y\,X\,\delta-4\,\frac{S\,X^2\,\kappa}{\alpha}-S\,{V_y}^2-S\,X^4\right)\alpha^2$$
$$+(-2\,S\,X^2\,\kappa-4\,S\,X\,\delta\,\kappa+4\,S\,X\,\delta\,\theta+2\,S\,V_y\,\theta+2\,S\,X^2\,\theta-2\,S\,V_y\,\kappa)\,\alpha+1-S\,\kappa^2$$
$$+2\,S\,\kappa\,\theta-S\,\theta^2+\%1\,M_2-4\,S\,\alpha^2\,\delta\,M_3-S\,\alpha^2\,M_4\Bigg)-$$
$$((-4\,S\,\alpha^2\,\delta\,V_y+(-4\,S\,\delta\,\kappa+4\,S\,\delta\,\theta)\,\alpha)\,M_2+\%1\,M_3-4\,S\,\alpha^2\,\delta\,M_4-S\,\alpha^2\,M_5)^2\bigg/\Bigg($$
$$\left(-4\,S\,X^3\,\delta-2\,S\,V_y\,X^2-4\,S\,V_y\,X\,\delta-4\,\frac{S\,X^2\,\kappa}{\alpha}-S\,{V_y}^2-S\,X^4\right)\alpha^2$$
$$+(-2\,S\,X^2\,\kappa-4\,S\,X\,\delta\,\kappa+4\,S\,X\,\delta\,\theta+2\,S\,V_y\,\theta+2\,S\,X^2\,\theta-2\,S\,V_y\,\kappa)\,\alpha+1-S\,\kappa^2$$
$$+2\,S\,\kappa\,\theta-S\,\theta^2+\%1\,M_2-4\,S\,\alpha^2\,\delta\,M_3-S\,\alpha^2\,M_4\Bigg)^2$$
$$\%1:=\left(-4\,\frac{S\,\kappa}{\alpha}-2\,S\,V_y\right)\alpha^2+(-2\,S\,\kappa+2\,S\,\theta)\,\alpha$$

```
> simplify("): "/(S*alpha): factor(subs(alpha=0,"));
```
dominant part in alpha

$$2\,\frac{3\,\kappa\,M_4+2\,\delta\,M_3\,\kappa-3\,\kappa\,{M_2}^2-\theta\,M_4-2\,\delta\,M_3\,\theta+\theta\,{M_2}^2}{-1+S\,\kappa^2-2\,S\,\kappa\,\theta+S\,\theta^2}$$

```
> factor(subs({M[3]=0,M[4]=3*M[2]^2},"));
```

$$4\,\frac{{M_2}^2\,(3\,\kappa-\theta)}{-1+S\,\kappa^2-2\,S\,\kappa\,\theta+S\,\theta^2}$$

!!!!!!!!!!!!!!!!!!!!!!!!! Equation 10a !!!!!!!!!!!!!!!!!!!!!!!
(neglecting s and kappa relative to 1 and dividing by 2 to account for sex-limited expression of the trait)

```
> Delta[Vx]:=(1/2)*(alpha*s/theta^2)*subs({S=0,kappa=alpha*(x-y
  )^2,M[2]=V[x]},")+mu[x];
```

$$\Delta_{Vx}:=-2\,\frac{s\,\alpha\,{V_x}^2\,(3\,\alpha\,(x-y)^2-\theta)}{\theta^2}+\mu_x$$

Derivation of equations 2b and 10b from equations 9

The proportion of males compatible with female trait y is (using X=x-x_mean and delta=x_mean-y_mean
so that x-y_mean=X+delta)

```
> P:=1-alpha*(V[y]+(X+delta)^2);
```

$$P := 1 - \alpha\,(V_y + (X + \delta)^2)$$

```
> expand(");
```

$$1 - \alpha\,V_y - \alpha\,X^2 - 2\,\alpha\,X\,\delta - \alpha\,\delta^2$$

```
> P:=subs(delta^2=kappa/alpha,");
```

$$P := 1 - \alpha\,V_y - \alpha\,X^2 - 2\,\alpha\,X\,\delta - \kappa$$

Male fitness function (eq.1b) is the integral over X of

```
> W[y]:=(1-alpha*(X-Y+delta)^2)*(1-B*P);
```

$$W_y := (1 - \alpha\,(X - Y + \delta)^2)\,(1 - B\,(1 - \alpha\,V_y - \alpha\,X^2 - 2\,\alpha\,X\,\delta - \kappa))$$

```
> expand("):collect(",delta);
```

$$\begin{aligned}&-2\,\alpha^2\,\delta^3\,B\,X + (\alpha\,B - \alpha + 4\,\alpha^2\,Y\,B\,X - \alpha^2\,B\,V_y - \alpha\,B\,\kappa - 5\,\alpha^2\,X^2\,B)\,\delta^2 + (-4\,\alpha^2\,X^3\,B\\&\quad + 2\,\alpha^2\,Y\,B\,V_y - 2\,\alpha\,X + 2\,\alpha\,Y - 2\,\alpha\,Y\,B + 6\,\alpha^2\,X^2\,Y\,B + 4\,B\,\alpha\,X - 2\,\alpha^2\,Y^2\,B\,X\\&\quad - 2\,\alpha\,X\,B\,\kappa + 2\,\alpha\,Y\,B\,\kappa - 2\,\alpha^2\,X\,B\,V_y)\,\delta + 1 + 2\,B\,\alpha\,X^2 - \alpha\,X^2 - \alpha\,Y^2 - \alpha\,X^2\,B\,\kappa\\&\quad + 2\,\alpha\,X\,Y - B + B\,\alpha\,V_y + B\,\kappa + \alpha\,Y^2\,B + 2\,\alpha^2\,X\,Y\,B\,V_y - \alpha^2\,X^2\,B\,V_y - \alpha^2\,X^4\,B\\&\quad + 2\,\alpha\,X\,Y\,B\,\kappa - 2\,\alpha\,X\,Y\,B + 2\,\alpha^2\,X^3\,Y\,B - \alpha\,Y^2\,B\,\kappa - \alpha^2\,Y^2\,B\,V_y - \alpha^2\,Y^2\,B\,X^2\end{aligned}$$

inregrating over X:

```
> subs(X^2=V[x],"): subs(X=0,");
```

this is because
Integral (x-x_mean)^2 p(x) dx=V[x] and Integral (x-x_mean) p(x) dx = 0.

$$\begin{aligned}&1 + (\alpha\,B - \alpha - \alpha^2\,B\,V_y - \alpha\,B\,\kappa - 5\,\alpha^2\,V_x\,B)\,\delta^2\\&\quad + (2\,\alpha^2\,Y\,B\,V_y + 2\,\alpha\,Y - 2\,\alpha\,Y\,B + 6\,\alpha^2\,V_x\,Y\,B + 2\,\alpha\,Y\,B\,\kappa)\,\delta + 2\,B\,\alpha\,V_x - \alpha\,V_x - \alpha\,Y^2\\&\quad - \alpha\,V_x\,B\,\kappa - B + B\,\alpha\,V_y + B\,\kappa + \alpha\,Y^2\,B - \alpha^2\,V_x\,B\,V_y - \alpha\,Y^2\,B\,\kappa - \alpha^2\,Y^2\,B\,V_y - \alpha^2\,Y^2\,B\,V_x\end{aligned}$$

Male fitness function defined by 1b

```
> W[y]:=expand(subs({delta^2=kappa/alpha,delta^3=delta*kappa/al
  pha},"));
```

$$\begin{aligned}W_y &:= 1 + 2\,B\,\kappa - \kappa - \kappa\,\alpha\,B\,V_y - \kappa^2\,B - 6\,\alpha\,V_x\,B\,\kappa + 2\,\alpha^2\,Y\,\delta\,B\,V_y + 2\,\alpha\,Y\,\delta - 2\,\alpha\,Y\,\delta\,B\\&\quad + 6\,\delta\,\alpha^2\,V_x\,Y\,B + 2\,\alpha\,Y\,\delta\,B\,\kappa + 2\,B\,\alpha\,V_x - \alpha\,V_x - \alpha\,Y^2 - B + B\,\alpha\,V_y + \alpha\,Y^2\,B - \alpha^2\,V_x\,B\,V_y\\&\quad - \alpha\,Y^2\,B\,\kappa - \alpha^2\,Y^2\,B\,V_y - \alpha^2\,Y^2\,B\,V_x\end{aligned}$$

coefficients a_i in fitness function expansion (8):

```
> A:=i->collect(expand(subs(Y=0,diff(W[y],Y$i)/i!)),alpha);
```

$$A := i \rightarrow \mathrm{collect}\left(\mathrm{expand}\left(\mathrm{subs}\left(Y = 0, \frac{\mathrm{diff}(W_y, Y\,\$\,i)}{i!}\right)\right), \alpha\right)$$

```
> subs(Y=0,W[y]): a[0]:=collect(expand("),alpha);
```

$$a_0 := -\alpha^2 V_x B V_y + (-\kappa B V_y - 6 V_x B \kappa + 2 B V_x - V_x + B V_y)\,\alpha + 1 + 2 B \kappa - \kappa - \kappa^2 B - B$$

```
> A(1): a[1]:=";
```

$$a_1 := (6 \delta V_x B + 2 \delta B V_y)\,\alpha^2 + (2 \delta B \kappa + 2 \delta - 2 \delta B)\,\alpha$$

```
> A(2): a[2]:=";
```

$$a_2 := (-B V_y - B V_x)\,\alpha^2 + (-1 + B - B \kappa)\,\alpha$$

```
> A(3): a[3]:=";
```

$$a_3 := 0$$

```
> A(4): a[4]:=";
```

$$a_4 := 0$$

Denominator of 9a:

```
> Denom:=a[0]+sum(a[i]*M[i],i=2..4);
```

$$Denom := -\alpha^2 V_x B V_y + (-\kappa B V_y - 6 V_x B \kappa + 2 B V_x - V_x + B V_y)\,\alpha + 1 + 2 B \kappa - \kappa - \kappa^2 B - B + ((-B V_y - B V_x)\,\alpha^2 + (-1 + B - B \kappa)\,\alpha)\,M_2$$

Numerator of 9a

```
> Numer1:=sum(a[i]*M[i+1],i=1..4);
```

$$Numer1 := ((6 \delta V_x B + 2 \delta B V_y)\,\alpha^2 + (2 \delta B \kappa + 2 \delta - 2 \delta B)\,\alpha)\,M_2 + ((-B V_y - B V_x)\,\alpha^2 + (-1 + B - B \kappa)\,\alpha)\,M_3$$

```
>    Equation 9a
> subs(delta^2=kappa/alpha, Numer1/Denom);
```

$$(((6 \delta V_x B + 2 \delta B V_y)\,\alpha^2 + (2 \delta B \kappa + 2 \delta - 2 \delta B)\,\alpha)\,M_2 + ((-B V_y - B V_x)\,\alpha^2 + (-1 + B - B \kappa)\,\alpha)\,M_3) \Big/ (-\alpha^2 V_x B V_y + (-\kappa B V_y - 6 V_x B \kappa + 2 B V_x - V_x + B V_y)\,\alpha + 1 + 2 B \kappa - \kappa - \kappa^2 B - B + ((-B V_y - B V_x)\,\alpha^2 + (-1 + B - B \kappa)\,\alpha)\,M_2)$$

```
> simplify(");
```

$$-\alpha\,(2 M_2 \alpha \delta B V_y + 2 \delta M_2 - 2 M_2 \delta B + 6 M_2 \delta \alpha V_x B + 2 M_2 \delta B \kappa - M_3 + M_3 B - M_3 B \kappa - M_3 \alpha B V_y - M_3 \alpha V_x B) \Big/ (\alpha^2 V_x B V_y + \kappa \alpha B V_y + 6 \alpha V_x B \kappa - 2 B \alpha V_x + \alpha V_x - B \alpha V_y - 1 - 2 B \kappa + \kappa + \kappa^2 B + B + M_2 \alpha - M_2 \alpha B + M_2 \alpha B \kappa + M_2 \alpha^2 B V_y + M_2 \alpha^2 V_x B)$$

```
> "/(alpha);
```

$$-(2 M_2 \alpha \delta B V_y + 2 \delta M_2 - 2 M_2 \delta B + 6 M_2 \delta \alpha V_x B + 2 M_2 \delta B \kappa - M_3 + M_3 B - M_3 B \kappa - M_3 \alpha B V_y - M_3 \alpha V_x B) \Big/ (\alpha^2 V_x B V_y + \kappa \alpha B V_y + 6 \alpha V_x B \kappa - 2 B \alpha V_x + \alpha V_x - B \alpha V_y - 1 - 2 B \kappa + \kappa + \kappa^2 B + B + M_2 \alpha - M_2 \alpha B + M_2 \alpha B \kappa + M_2 \alpha^2 B V_y$$

$+ M_2 \alpha^2 V_x B)$

Dominant part (in alpha)

```
> factor(subs(alpha=0,"));
```

$$-\frac{2\,\delta\,M_2 - M_3}{\kappa - 1}$$

Neglecting 3rd moment

```
> factor(subs({M[3]=0,M[4]=3*M[2]^2},"));
```

$$-2\,\frac{\delta\,M_2}{\kappa - 1}$$

!!!!!!!!!!!!!!!!!!!!!!!!! !! Equation (2b). !!!!!!!!!!!!!!!!!!

(neglecting kappa relative to 1 and dividing by 2 to account forsex-limited expression of trait)

```
> Delta[y]:=subs({kappa=0,M[2]=V[y],delta=x-y},alpha*"/2);
```

$$\Delta_y := \alpha\,(x - y)\,V_y$$

Numerator in 9b

```
> Numer2:=a[1]*M[3]+sum(a[i]*(M[i+2]-M[i]*M[2]),i=2..4);
```

$$\mathit{Numer2} := ((6\,\delta\,V_x\,B + 2\,\delta\,B\,V_y)\,\alpha^2 + (2\,\delta\,B\,\kappa + 2\,\delta - 2\,\delta\,B)\,\alpha)\,M_3 + ((-B\,V_y - B\,V_x)\,\alpha^2 + (-1 + B - B\,\kappa)\,\alpha)\,(M_4 - M_2^{\,2})$$

Equation 9b

```
> subs(delta^2=kappa/alpha, Numer2/Denom-(Numer1/Denom)^2);
```

$$(((6\,\delta\,V_x\,B + 2\,\delta\,B\,V_y)\,\alpha^2 + (2\,\delta\,B\,\kappa + 2\,\delta - 2\,\delta\,B)\,\alpha)\,M_3 + \%1\,(M_4 - M_2^{\,2})) \Big/ (-\alpha^2\,V_x\,B\,V_y + (-\kappa\,B\,V_y - 6\,V_x\,B\,\kappa + 2\,B\,V_x - V_x + B\,V_y)\,\alpha + 1 + 2\,B\,\kappa - \kappa - \kappa^2\,B - B + \%1\,M_2) - (((6\,\delta\,V_x\,B + 2\,\delta\,B\,V_y)\,\alpha^2 + (2\,\delta\,B\,\kappa + 2\,\delta - 2\,\delta\,B)\,\alpha)\,M_2 + \%1\,M_3)^2 \Big/ (-\alpha^2\,V_x\,B\,V_y + (-\kappa\,B\,V_y - 6\,V_x\,B\,\kappa + 2\,B\,V_x - V_x + B\,V_y)\,\alpha + 1 + 2\,B\,\kappa - \kappa - \kappa^2\,B - B + \%1\,M_2)^2$$

$$\%1 := (-B\,V_y - B\,V_x)\,\alpha^2 + (-1 + B - B\,\kappa)\,\alpha$$

```
> simplify("): "/alpha: factor(subs(alpha=0,"));
```

$$-\frac{2\,\delta\,M_3 - M_4 + M_2^{\,2}}{\kappa - 1}$$

```
> factor(subs({M[3]=0,M[4]=3*M[2]^2},alpha*"));
```

$$2\,\frac{\alpha\,M_2^{\,2}}{\kappa - 1}$$

!!!!!!!!!!!!!!!!!!!!!!!! Equation 10b !!!!!!!!!!!!!!!!!!

(neglecting kappa relative to 1 and dividing by 2 to account for sex-limited expression of trait)

```
> Delta[Vy]:=subs({kappa=0,M[2]=V[y]},"/2)+mu[y];
```

$$\Delta_{Vy} := -V_y^2\,\alpha + \mu_y$$

```
>
```